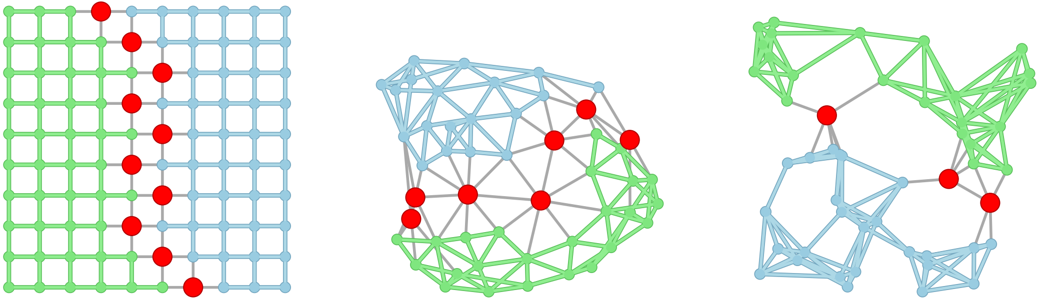
Reweighted Spectral Partitioning Works

A Simple Algorithm for Vertex Separators in Special Graph Classes

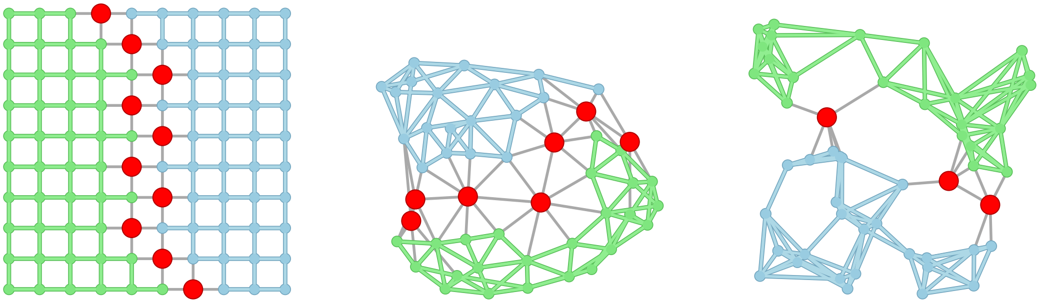
Jack Spalding-Jamieson

Balanced Vertex Separators



Vertex Separator: Small set of vertices whose removal disconnects into small components.

Balanced Vertex Separators

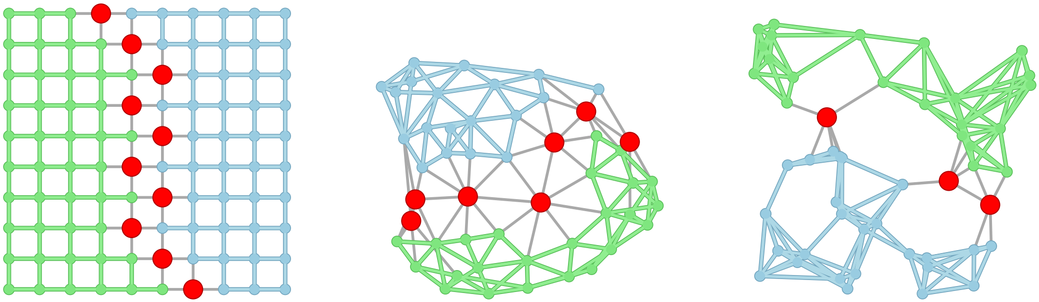


Vertex Separator: Small set of vertices whose removal disconnects into small components.

Theorem (Planar Separator Theorem)

For a planar graph G of n vertices, there is a subset S of $\mathcal{O}\left(\sqrt{n}\right)$ vertices so that every connected component of $G - S$ has at most $\frac{2}{3}n$ vertices.

Balanced Vertex Separators

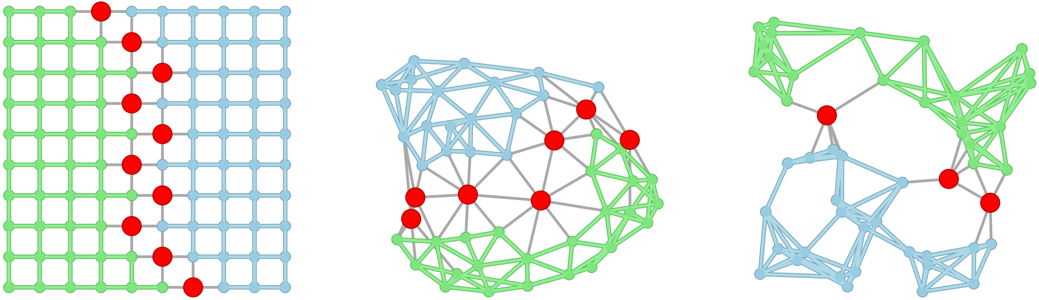


Vertex Separator: Small set of vertices whose removal disconnects into small components.

Theorem (Planar Separator Theorem)

For a planar graph G of n vertices, there is a subset S of $\mathcal{O}\left(\sqrt{n}\right)$ vertices so that every connected component of $G - S$ has at most $\frac{2}{3}n$ vertices. S can be found in $\mathcal{O}(n)$ time.

Balanced Vertex Separators



Vertex Separator: Small set of vertices whose removal disconnects into small components.

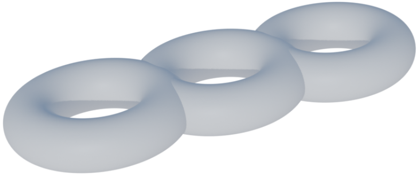
Theorem (Planar Separator Theorem)

For a planar graph G of n vertices, there is a subset S of $\mathcal{O}\left(\sqrt{n}\right)$ vertices so that every connected component of $G - S$ has at most $\frac{2}{3}n$ vertices. S can be found in $\mathcal{O}(n)$ time.

Small separator = many fast algorithms!

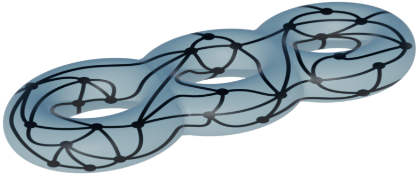
Other Separator Theorems (1)

Genus-g graph: Embeddable on genus-g surface without crossings.



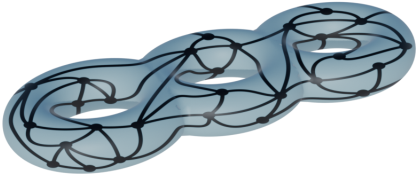
Other Separator Theorems (1)

Genus-g graph: Embeddable on genus-g surface without crossings.



Other Separator Theorems (1)

Genus-g graph: Embeddable on genus-g surface without crossings.

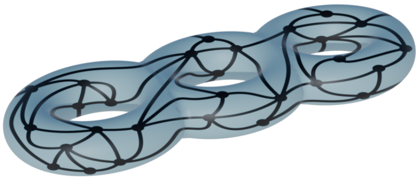


Theorem (Genus-g Separator Theorem)

Genus g graph: Separator size $\mathcal{O}\left(\sqrt{gn}\right)$.

Other Separator Theorems (1)

Genus-g graph: Embeddable on genus-g surface without crossings.

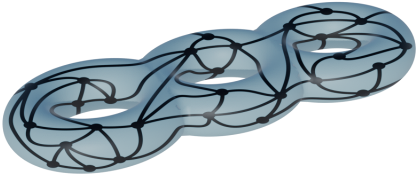


Theorem (Genus-g Separator Theorem)

Genus g graph: Separator size $\mathcal{O}\left(\sqrt{gn}\right)$. Can be found in $\mathcal{O}(n)$ time, **if a surface embedding is provided**.

Other Separator Theorems (1)

Genus-g graph: Embeddable on genus-g surface without crossings.



Theorem (Genus-g Separator Theorem)

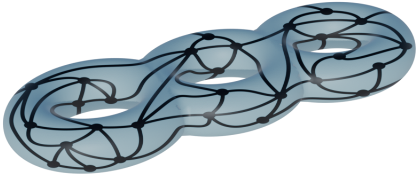
Genus g graph: Separator size $\mathcal{O}\left(\sqrt{gn}\right)$. Can be found in $\mathcal{O}(n)$ time, **if a surface embedding is provided**.

Theorem (K_h-minor-free Separator Theorem)

K_h-minor-free graph: Separator size $\mathcal{O}\left(h\sqrt{n}\right)$.

Other Separator Theorems (1)

Genus-g graph: Embeddable on genus-g surface without crossings.



Theorem (Genus-g Separator Theorem)

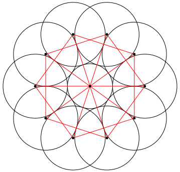
Genus g graph: Separator size $\mathcal{O}\left(\sqrt{gn}\right)$. Can be found in $\mathcal{O}(n)$ time, **if a surface embedding is provided**.

Theorem (K_h-minor-free Separator Theorem)

K_h-minor-free graph: Separator size $\mathcal{O}\left(h\sqrt{n}\right)$. Can be found in $\mathcal{O}(n^2)$ time, **provided that** h **is constant**.

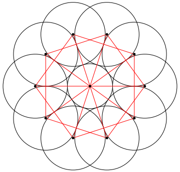
Other Separator Theorems (2)

k-ply d-dimensional sphere-intersection graph



Other Separator Theorems (2)

k-ply d-dimensional sphere-intersection graph



Theorem (k-ply d-dimensional ball-intersection Separator Theorem [MTTV97])

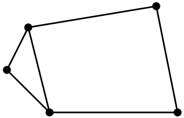
*k-ply d-dimensional ball-intersection graph: Separator size $\mathcal{O}\left(dk^{\frac{1}{d}}n^{1-\frac{1}{d}}\right)$. Can be found in $\mathcal{O}(f(d) + nd^2)$ time, for a function f, **if the points are provided**.*

Theorem (Improved k-ply d-dimensional ball-intersection Separator Theorem [New, Side-Result])

*k-ply d-dimensional ball-intersection graph: Separator size $\mathcal{O}\left(\sqrt{\min\{d, \log \Delta\}}k^{\frac{1}{d}}n^{1-\frac{1}{d}}\right)$. Can be found in polynomial time, **if the points are provided**.*

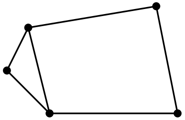
Other Separator Theorems (3)

d-dimensional k-NN graph: For n points in the plane, add an edge to each point's top k-nearest neighbours.



Other Separator Theorems (3)

d-dimensional k-NN graph: For n points in the plane, add an edge to each point's top k-nearest neighbours.



Theorem (d-dimensional k-NN Separator Theorem [MTTV97])

d-dimensional k-NN graph: Separator size $\mathcal{O}\left(dk^{\frac{1}{d}}n^{1-\frac{1}{d}}\right)$. Can be found in $\mathcal{O}(f(d) + nd^2)$ time, for a function f, ***if the points are provided**.*

Theorem (Improved d-dimensional k-NN Separator Theorem [New, Side-Result])

d-dimensional k-NN graph with max degree Δ: Separator size $\mathcal{O}\left(\sqrt{\min\{d, \log \Delta\}}k^{\frac{1}{d}}n^{1-\frac{1}{d}}\right)$. Can be found in polynomial time, ***if the points are provided**.*

No additional information!

In practice: Given graph, no specialized info. Want theoretically sound guarantees of small separators.

Three broad approaches:

No additional information!

In practice: Given graph, no specialized info. Want theoretically sound guarantees of small separators.

Three broad approaches:

- **Multiple algorithms**: Only works for certain graph classes, time complexity might depend on class.

No additional information!

In practice: Given graph, no specialized info. Want theoretically sound guarantees of small separators.

Three broad approaches:

- **Multiple algorithms**: Only works for certain graph classes, time complexity might depend on class.
- **Approximation algorithm**: Works for *all* graphs, but approximation ratio is worst-case.

No additional information!

In practice: Given graph, no specialized info. Want theoretically sound guarantees of small separators.

Three broad approaches:

- **Multiple algorithms**: Only works for certain graph classes, time complexity might depend on class.
- **Approximation algorithm**: Works for *all* graphs, but approximation ratio is worst-case.
- **One algorithm, multiple proofs**: Unspecialized algorithm, extra proofs for special inputs.

No additional information!

In practice: Given graph, no specialized info. Want theoretically sound guarantees of small separators.

Three broad approaches:

- **Multiple algorithms**: Only works for certain graph classes, time complexity might depend on class.
- **Approximation algorithm**: Works for *all* graphs, but approximation ratio is worst-case.
- **One algorithm, multiple proofs**: Unspecialized algorithm, extra proofs for special inputs.

This talk is about the third kind!

No additional information!

In practice: Given graph, no specialized info. Want theoretically sound guarantees of small separators.

Three broad approaches:

- **Multiple algorithms**: Only works for certain graph classes, time complexity might depend on class.
- **Approximation algorithm**: Works for *all* graphs, but approximation ratio is worst-case.
- **One algorithm, multiple proofs**: Unspecialized algorithm, extra proofs for special inputs.

This talk is about the third kind!

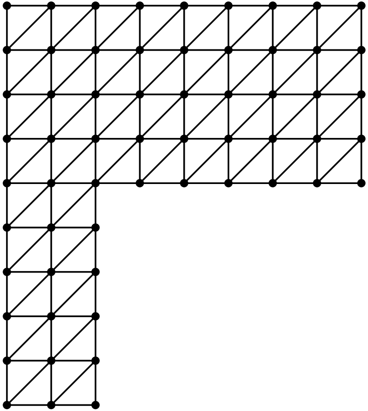
The algorithm we consider: **Reweighted Spectral Partitioning**.

Results

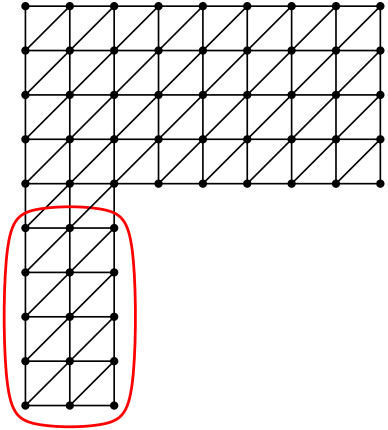
Graph class	This work	Previous work
Genus-g	$\mathcal{O}(\min\{(\log g)^2\sqrt{gn}, \boldsymbol{\log \Delta \sqrt{gn}}\})$	$\mathcal{O}(\min\{\boldsymbol{(\log g)\sqrt{gn}}, \text{poly}(\Delta)\sqrt{gn}\})$
K_h-minor-free	$\mathcal{O}(\min\{\log h, \sqrt{\log \Delta}\}(h \log h \log\log h)\sqrt{n})$	$\boldsymbol{\mathcal{O}((\log h)h\sqrt{n})}$
k-ply ball-intersection in $\mathbb{R}^d$	$\boldsymbol{\mathcal{O}(\sqrt{\log \Delta} \cdot k^{1/d} n^{1-1/d})}$	$\mathcal{O}(\sqrt{\log n} \cdot dk^{1/d} n^{1-1/d})$
k-nearest-neighbour in $\mathbb{R}^d$	$\boldsymbol{\mathcal{O}(\sqrt{\log \Delta} \cdot k^{1/d} n^{1-1/d})}$	$\mathcal{O}(\sqrt{\log n} \cdot dk^{1/d} n^{1-1/d})$

Reweighted spectral partitioning separator size guarantees (via this work) vs. previous algorithms.

Separators, Expansion, and Cuts

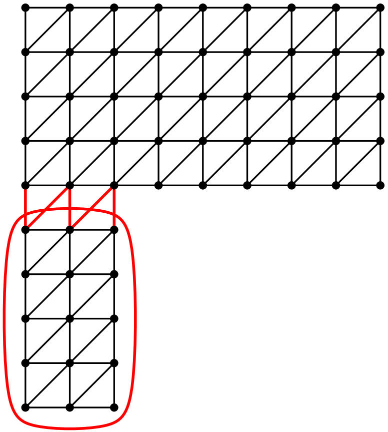


Separators, Expansion, and Cuts



Want small boundary to area ratio

Separators, Expansion, and Cuts

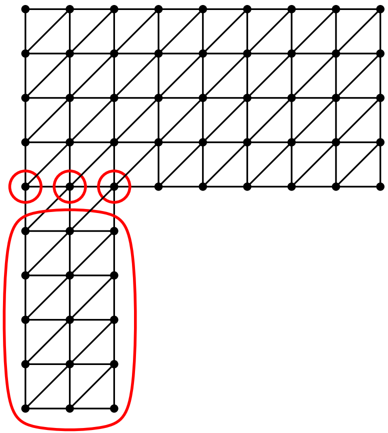


Want small boundary to area ratio

For a set $S \subset V$, **edge expansion** of S is:

$$\phi(S) := \frac{|E(S, S^c)|}{|S|}.$$

Separators, Expansion, and Cuts



Want small boundary to area ratio

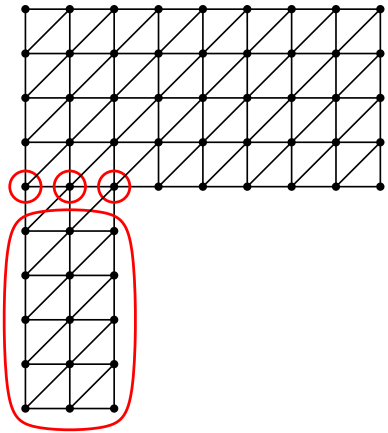
For a set $S \subset V$, **edge expansion** of S is:

$$\phi(S) := \frac{|E(S, S^c)|}{|S|}.$$

For a set $S \subset V$, **vertex expansion** of S is:

$$\psi(S) := \frac{|N(S) \cap S^c|}{|S|}.$$

Separators, Expansion, and Cuts



Want small boundary to area ratio

For a set $S \subset V$, **edge expansion** of S is:

$$\phi(S) := \frac{|E(S, S^c)|}{|S|}.$$

For a set $S \subset V$, **vertex expansion** of S is:

$$\psi(S) := \frac{|N(S) \cap S^c|}{|S|}.$$

Known algorithm:

For induced subgraph $H \subset G$, find cut S with $\psi(S) \leq \frac{\alpha}{n^{\varepsilon}}$
$\implies$ can get balanced vertex separator of size $O(\alpha n^{1-\varepsilon})$.

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

hard to compute, want to approximate

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \boldsymbol{\lambda_2}(G) \leq 2\phi(G)$$

easy to compute

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \leftarrow \lambda_2(G) \leq 2\phi(G)$$

algorithmic, generic!

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

Spectral partitioning algorithm:
Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \le \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \le \lambda_2(G) \le 2\phi(G)$$

Spectral partitioning algorithm:
Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Fact for planar graphs: $\phi(G) \lesssim \sqrt{\frac{\Delta}{n}}$.

$\lambda_2(G)$+Cheeger: $\phi(S) \lesssim \sqrt{\Delta\sqrt{\frac{\Delta}{n}}}$ (weak).

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

Spectral partitioning algorithm:
Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Fact for planar graphs: $\phi(G) \lesssim \sqrt{\frac{\Delta}{n}}$.

$\lambda_2(G)$+Cheeger: $\phi(S) \lesssim \sqrt{\Delta\sqrt{\frac{\Delta}{n}}}$ (weak).

First specialized proof:
Spectral Partitioning Works
by Daniel Spielman and Shang-Hua Teng.

Showed for planar graph, $\lambda_2(G) \lesssim \frac{\Delta}{n}$

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

Spectral partitioning algorithm:
Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Fact for planar graphs: $\phi(G) \lesssim \sqrt{\frac{\Delta}{n}}$.

$\lambda_2(G)$+Cheeger: $\phi(S) \lesssim \sqrt{\Delta\sqrt{\frac{\Delta}{n}}}$ (weak).

First specialized proof:
Spectral Partitioning Works
by Daniel Spielman and Shang-Hua Teng.

Showed for planar graph, $\lambda_2(G) \lesssim \frac{\Delta}{n}$

Now $\lambda_2(G)$+Cheeger $\implies \phi(S) \lesssim \frac{\Delta^{\frac{3}{2}}}{\sqrt{n}}$.

A Related Example: Spectral Partitioning of Graphs

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

Spectral partitioning algorithm:
Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Fact for planar graphs: $\phi(G) \lesssim \sqrt{\frac{\Delta}{n}}$.

$\lambda_2(G)$+Cheeger: $\phi(S) \lesssim \sqrt{\Delta\sqrt{\frac{\Delta}{n}}}$ (weak).

First specialized proof:
Spectral Partitioning Works
by Daniel Spielman and Shang-Hua Teng.

Showed for planar graph, $\lambda_2(G) \lesssim \frac{\Delta}{n}$

Now $\lambda_2(G)$+Cheeger $\implies \phi(S) \lesssim \frac{\Delta^{\frac{3}{2}}}{\sqrt{n}}$.

Similar results for many other classes (also in other works).

From Spectral Partitioning to **Reweighted** Spectral Partitioning

Edge expansion $\phi(G) := \min_{|S| \le \frac{n}{2}} \phi(S)$:
Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \le \lambda_2(G) \le 2\phi(G)$$

Spectral partitioning algorithm:
Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

From Spectral Partitioning to **Reweighted** Spectral Partitioning

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$: Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

Spectral partitioning algorithm: Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Vertex expansion $\psi(G) := \min_{|S| \leq \frac{n}{2}} \psi(S)$: Small = fast algorithms.

Max Reweighted Spec Gap ($\gamma^{(n)}$): A poly-time computable quantity.

From Spectral Partitioning to **Reweighted** Spectral Partitioning

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$: Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

Spectral partitioning algorithm: Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Vertex expansion $\psi(G) := \min_{|S| \leq \frac{n}{2}} \psi(S)$: Small = fast algorithms.

Max Reweighted Spec Gap $(\gamma^{(n)})$: A poly-time computable quantity.

Theorem (Cheeger-Style Inequality [Roc05, OTZ22, JPV22, KLT22])

For a graph G with max degree Δ,

$$\frac{\psi(G)^2}{\log \Delta} \lesssim \gamma^{(n)}(G) \lesssim \psi(G).$$

From Spectral Partitioning to **Reweighted** Spectral Partitioning

Edge expansion $\phi(G) := \min_{|S| \le \frac{n}{2}} \phi(S)$: Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \le \lambda_2(G) \le 2\phi(G)$$

Spectral partitioning algorithm: Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Vertex expansion $\psi(G) := \min_{|S| \le \frac{n}{2}} \psi(S)$: Small = fast algorithms.

Max Reweighted Spec Gap $(\gamma^{(n)})$: A poly-time computable quantity.

Theorem (Cheeger-Style Inequality [Roc05, OTZ22, JPV22, KLT22])

For a graph G with max degree Δ,

$$\frac{\psi(G)^2}{\log \Delta} \lesssim \gamma^{(n)}(G) \lesssim \psi(G).$$

hard to compute, want to approximate

From Spectral Partitioning to **Reweighted** Spectral Partitioning

Edge expansion $\phi(G) := \min_{|S| \le \frac{n}{2}} \phi(S)$: Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \le \lambda_2(G) \le 2\phi(G)$$

Spectral partitioning algorithm: Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Vertex expansion $\psi(G) := \min_{|S| \le \frac{n}{2}} \psi(S)$: Small = fast algorithms.

Max Reweighted Spec Gap $(\gamma^{(n)})$: A poly-time computable quantity.

Theorem (Cheeger-Style Inequality [Roc05, OTZ22, JPV22, KLT22])

For a graph G with max degree Δ,

$$\frac{\psi(G)^2}{\log \Delta} \lesssim \gamma^{(n)}(G) \lesssim \psi(G).$$

easy to compute

From Spectral Partitioning to **Reweighted** Spectral Partitioning

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$: Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

Spectral partitioning algorithm: Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Vertex expansion $\psi(G) := \min_{|S| \leq \frac{n}{2}} \psi(S)$: Small = fast algorithms.

Max Reweighted Spec Gap ($\gamma^{(n)}$): A poly-time computable quantity.

Theorem (Cheeger-Style Inequality [Roc05, OTZ22, JPV22, KLT22])

For a graph G with max degree Δ,

$$\frac{\psi(G)^2}{\log \Delta} \lesssim \leftarrow \gamma^{(n)}(G) \lesssim \psi(G).$$

algorithmic, generic!

From Spectral Partitioning to **Reweighted** Spectral Partitioning

Edge expansion $\phi(G) := \min_{|S| \leq \frac{n}{2}} \phi(S)$: Small = fast algorithms.

Fiedler Value (λ_2): A poly-time computable "spectral" quantity.

Theorem (Cheeger's inequality)

For a graph G with max degree Δ,

$$\frac{\phi(G)^2}{2\Delta} \leq \lambda_2(G) \leq 2\phi(G)$$

Spectral partitioning algorithm: Compute $\lambda_2(G)$, obtain S with $\phi(S)$ bounded.

Vertex expansion $\psi(G) := \min_{|S| \leq \frac{n}{2}} \psi(S)$: Small = fast algorithms.

Max Reweighted Spec Gap ($\gamma^{(n)}$): A poly-time computable quantity.

Theorem (Cheeger-Style Inequality [Roc05, OTZ22, JPV22, KLT22])

For a graph G with max degree Δ,

$$\frac{\psi(G)^2}{\log \Delta} \lesssim \gamma^{(n)}(G) \lesssim \psi(G).$$

Reweighted spectral partitioning algorithm: Compute $\gamma^{(n)}(G)$, obtain S with $\psi(S)$ bounded.

Refining Reweighted Spectral Partitioning

Theorem (Refined Cheeger-Style Inequality [New])

For a graph G with n vertices and maximum degree Δ,

$$\frac{\psi(G)^2}{\min\{\log \Delta, \alpha(G)^2\}} \lesssim \gamma^{(n)}(G) \lesssim \psi(G).$$

Refining Reweighted Spectral Partitioning

Theorem (Refined Cheeger-Style Inequality [New])

For a graph G with n vertices and maximum degree Δ,

$$\frac{\psi(G)^2}{\min\{\log \Delta, \alpha(G)^2\}} \lesssim \gamma^{(n)}(G) \lesssim \psi(G).$$

$\alpha(G)$ is the *worst-case modulus of padded decomposition for vertex-weighted shortest-path metrics* over G.

Refining Reweighted Spectral Partitioning

Theorem (Refined Cheeger-Style Inequality [New])

For a graph G with n vertices and maximum degree Δ,

$$\frac{\psi(G)^2}{\min\{\log \Delta, \alpha(G)^2\}} \lesssim \gamma^{(n)}(G) \lesssim \psi(G).$$

$\alpha(G)$ is the ~~*worst-case modulus of padded decomposition for vertex-weighted shortest-path metrics over G.*~~ **intrinsic dimension** of G.

Refining Reweighted Spectral Partitioning

Theorem (Refined Cheeger-Style Inequality [New])

For a graph G with n vertices and maximum degree Δ,

$$\frac{\psi(G)^2}{\min\{\log \Delta, \alpha(G)^2\}} \lesssim \gamma^{(n)}(G) \lesssim \psi(G).$$

$\alpha(G)$ is the ~~*worst-case modulus of padded decomposition for vertex-weighted shortest-path metrics over G.*~~ **intrinsic dimension** of G.

E.g. G planar $\implies \alpha(G) \in \mathcal{O}(1)$.

Refining Reweighted Spectral Partitioning

Theorem (Refined Cheeger-Style Inequality [New])

For a graph G with n vertices and maximum degree Δ,

$$\frac{\psi(G)^2}{\min\{\log\Delta, \alpha(G)^2\}} \lesssim \gamma^{(n)}(G) \lesssim \psi(G).$$

$\alpha(G)$ is the ~~*worst-case modulus of padded decomposition for vertex-weighted shortest-path metrics over G.*~~ **intrinsic dimension** of G.

E.g. G planar $\implies \alpha(G) \in \mathcal{O}(1)$.

Reweighted Spectral Partitioning **Works**

Reweighted spectral partitioning works: Direct class-specific upper bounds for $\gamma^{(n)}(G)$.

Graph class	$\gamma^{(n)} \leq \gamma^{(1)} \lesssim$
Planar	$\frac{1}{n}$
Genus-g	$\frac{g \min\{(\log g)^2, \log \Delta\}}{n}$
K_h-minor-free	$\frac{(h \log h \log \log h)^2}{n}$

Reweighted Spectral Partitioning Works

Reweighted spectral partitioning works: Direct class-specific upper bounds for $\gamma^{(n)}(G)$.

Graph class	$\gamma^{(n)} \leq \gamma^{(1)} \lesssim$
Planar	$\frac{1}{n}$
Genus-g	$\frac{g \min\{(\log g)^2, \log \Delta\}}{n}$
K_h-minor-free	$\frac{(h \log h \log \log h)^2}{n}$

Graph class	$\gamma^{(n)} \leq \gamma^{(d)} \lesssim$
(d-dim) k-ply ball-intersection	$\left(\frac{k}{n}\right)^{\frac{2}{d}}$
(d-dim) k-NN graph	$\left(\frac{k}{n}\right)^{\frac{2}{d}}$

Reweighted Spectral Partitioning **Works**

Reweighted spectral partitioning works: Direct class-specific upper bounds for $\gamma^{(n)}(G)$.

Graph class	$\gamma^{(n)} \leq \gamma^{(1)} \lesssim$
Planar	$\frac{1}{n}$
Genus-g	$\frac{g \min\{(\log g)^2, \log \Delta\}}{n}$
K_h-minor-free	$\frac{(h \log h \log\log h)^2}{n}$

Graph class	$\gamma^{(n)} \leq \gamma^{(d)} \lesssim$
(d-dim) k-ply ball-intersection	$\left(\frac{k}{n}\right)^{\frac{2}{d}}$
(d-dim) k-NN graph	$\left(\frac{k}{n}\right)^{\frac{2}{d}}$

E.g. G planar $\implies \gamma^{(n)}(G) \lesssim \frac{1}{n} \implies \psi(S) \lesssim \frac{1}{\sqrt{n}} \implies$ reproduces planar separator theorem!

Intuition for $\gamma^{(n)}(G)$

Definition

For a graph G, define:

$$\gamma^{(d)}(G) := \min_{\substack{f:V\to\mathbb{R}^d \\ y:V\to\mathbb{R}_{\geq 0}}} \frac{\sum_{v\in V} y(v)}{\sum_{x\in V} ||f(x)||_2^2}$$

$$\text{subject to} \quad \sum_{v\in V} f(v) = \bar{0}$$

$$y(u) + y(v) \geq ||f(u) - f(v)||_2^2 \quad \forall uv \in E$$

Intuition for $\gamma^{(n)}(G)$

Definition

For a graph G, define:

$$\gamma^{(d)}(G) := \min_{\substack{f:V\to\mathbb{R}^d \\ y:V\to\mathbb{R}_{\geq 0}}} \frac{\sum_{v\in V} y(v)}{\sum_{x\in V} ||f(x)||_2^2}$$

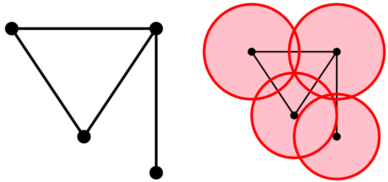
subject to

$$\sum_{v\in V} f(v) = \bar{0}$$

$$y(u)+y(v) \geq ||f(u)-f(v)||_2^2 \quad \forall uv\in E$$

Loose interpretation:

- For $v\in V$: Create a ball in $\mathbb{R}^d$ centred at $f(v)$, radius $\approx \sqrt{y(v)}$.



Intuition for $\gamma^{(n)}(G)$

Definition

For a graph G, define:

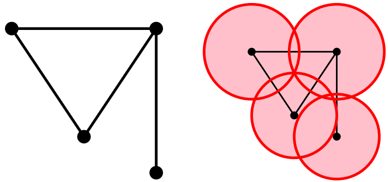
$$\gamma^{(d)}(G) \quad := \quad \min_{\substack{f:V\to\mathbb{R}^d \\ y:V\to\mathbb{R}_{\geq 0}}} \quad \frac{\sum_{v\in V} y(v)}{\sum_{x\in V} ||f(x)||_2^2}$$

$$\text{subject to} \qquad \sum_{v\in V} f(v) \;=\; \bar{0}$$

$$y(u) + y(v) \;\geq\; ||f(u) - f(v)||_2^2 \quad \forall uv \in E$$

Loose interpretation:

- For $v \in V$: Create a ball in $\mathbb{R}^d$ centred at $f(v)$, radius $\approx \sqrt{y(v)}$.
- Minimize sum of squared radii under normalization constraints.



Intuition for $\gamma^{(n)}(G)$

Definition

For a graph G, define:

$$\gamma^{(d)}(G) := \min_{\substack{f:V\to\mathbb{R}^d \\ y:V\to\mathbb{R}_{\geq 0}}} \frac{\sum_{v\in V} y(v)}{\sum_{x\in V} ||f(x)||_2^2}$$

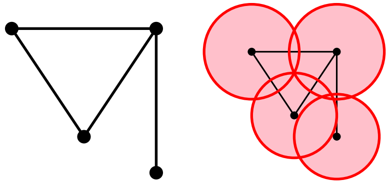
subject to

$$\sum_{v\in V} f(v) = \bar{0}$$

$$y(u) + y(v) \geq ||f(u) - f(v)||_2^2 \quad \forall uv \in E$$

Loose interpretation:

- For $v \in V$: Create a ball in $\mathbb{R}^d$ centred at $f(v)$, radius $\approx \sqrt{y(v)}$.
- Minimize sum of squared radii under normalization constraints.



Intuition for $\gamma^{(n)}(G)$

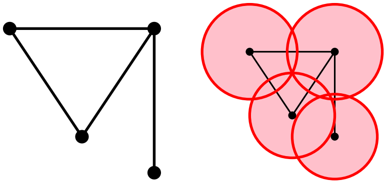
Definition

For a graph G, define:

$$
\begin{aligned}
\gamma^{(d)}(G) \quad := \quad & \min_{\substack{f: V \to \mathbb{R}^d \\ y: V \to \mathbb{R}_{\geq 0}}} \quad \frac{\sum_{v \in V} y(v)}{\sum_{x \in V} \|f(x)\|_2^2} \\
& \text{subject to} \quad \sum_{v \in V} f(v) \quad = \quad \bar{0} \\
& \qquad\qquad y(u) + y(v) \quad \geq \quad \|f(u) - f(v)\|_2^2 \quad \forall uv \in E
\end{aligned}
$$

Loose interpretation:

- For $v \in V$: Create a ball in $\mathbb{R}^d$ centred at $f(v)$, radius $\approx \sqrt{y(v)}$.
- Minimize sum of squared radii under normalization constraints.
- Constraint: Adjacent balls *must* intersect.



Expanding the Cheeger-Style Inequality

Theorem (Refined Cheeger-Style Inequality, expanded)

For a graph G with n vertices and maximum degree Δ,

$$\frac{\psi(G)^2}{\min\{\log \Delta, [\alpha(G)]^2\}} \lesssim \frac{\gamma^{(1)}(G)}{\min\{\log \Delta, [\alpha(G)]^2\}} \lesssim \gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \psi(G).$$

Reminder: $\gamma^{(n)}$ is the poly-time computable quantity (it is an SDP).

Expanding the Cheeger-Style Inequality

Theorem (Refined Cheeger-Style Inequality, expanded)

For a graph G with n vertices and maximum degree Δ,

$$\frac{\psi(G)^2}{\min\{\log \Delta, [\alpha(G)]^2\}} \lesssim \frac{\gamma^{(1)}(G)}{\min\{\log \Delta, [\alpha(G)]^2\}} \lesssim \gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \psi(G).$$

Lemma (OTZ22)

For a graph G,

$$\psi(G)^2 \lesssim \gamma^{(1)}(G) \lesssim \psi(G).$$

Expanding the Cheeger-Style Inequality

Theorem (Refined Cheeger-Style Inequality, expanded)

For a graph G with n vertices and maximum degree Δ,

$$\frac{\psi(G)^2}{\min\{\log\Delta, [\alpha(G)]^2\}} \lesssim \frac{\gamma^{(1)}(G)}{\min\{\log\Delta, [\alpha(G)]^2\}} \lesssim \gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \psi(G).$$

Lemma (OTZ22)

For a graph G,

$$\psi(G)^2 \lesssim \gamma^{(1)}(G) \lesssim \psi(G).$$

Lemma (Dimension-reduction step [KLT22])

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \log\Delta.$$

Expanding the Cheeger-Style Inequality

Theorem (Refined Cheeger-Style Inequality, expanded)

For a graph G with n vertices and maximum degree Δ,

$$\frac{\psi(G)^2}{\min\{\log\Delta, [\alpha(G)]^2\}} \lesssim \frac{\gamma^{(1)}(G)}{\min\{\log\Delta, [\alpha(G)]^2\}} \lesssim \gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \psi(G).$$

Lemma (OTZ22)

For a graph G,

$$\psi(G)^2 \lesssim \gamma^{(1)}(G) \lesssim \psi(G).$$

Lemma (Dimension-reduction step [KLT22])

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \log\Delta.$$

Method: Don't change sphere radii, use random projection on centres.

Refining the Cheeger-Style Inequality

Lemma (New)

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \alpha(G)^2.$$

Refining the Cheeger-Style Inequality

Lemma (New)

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \alpha(G)^2.$$

New method: Embeddings of shortest-path metrics!

Refining the Cheeger-Style Inequality

Lemma (New)

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \alpha(G)^2.$$

New method: Embeddings of shortest-path metrics!

Let $\omega(v) :=$ radius of v. Use ω-weighted SPs on G.

Refining the Cheeger-Style Inequality

Lemma (New)

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \alpha(G)^2.$$

New method: Embeddings of shortest-path metrics!

Let $\omega(v) :=$ radius of v. Use ω-weighted SPs on G.

Theorem (Rab08, BLR10, KR10)

For a metric space (X, d), a Monte Carlo algorithm can compute a **non-expansive** *embedding of d into the line with* **average** 2-**distortion** $\mathcal{O}(\alpha(X, d)^2)$.

Note: For a vertex-weighted shortest-path metric (X, d) on G, $\alpha(G) \leq \alpha(X, d)$.

Refining the Cheeger-Style Inequality

Lemma (New)

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \alpha(G)^2.$$

New method: Embeddings of shortest-path metrics!
Let $\omega(v) :=$ radius of v. Use ω-weighted SPs on G.

Theorem (Rab08, BLR10, KR10)

For a metric space (X, d), a Monte Carlo algorithm can compute a **non-expansive** *embedding of d into the line with* **average** 2-**distortion** $\mathcal{O}(\alpha(X, d)^2)$.

Note: For a vertex-weighted shortest-path metric (X, d) on G, $\alpha(G) \leq \alpha(X, d)$.
Ongoing follow-up work: This is now deterministic.

Refining the Cheeger-Style Inequality

Lemma (New)

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \alpha(G)^2.$$

New method: Embeddings of shortest-path metrics!

Let $\omega(v) :=$ radius of v. Use ω-weighted SPs on G.

Theorem (Rab08, BLR10, KR10, unpublished follow-up work)

For a graph G with vertex-weights $\omega : V \to \mathbb{R}_{\geq 0}$, a deterministic algorithm can compute a **non-expansive** *embedding of d_ω into the line with* **average** 2-**distortion** $\mathcal{O}(\alpha(G)^2)$.

Refining the Cheeger-Style Inequality

Lemma (New)

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \alpha(G)^2.$$

New method: Embeddings of shortest-path metrics!

Let $\omega(v) :=$ radius of v. Use ω-weighted SPs on G.

Theorem (Rab08, BLR10, KR10, unpublished follow-up work)

For a graph G with vertex-weights $\omega : V \to \mathbb{R}_{\geq 0}$, a deterministic algorithm can compute a **non-expansive** *embedding of d_ω into the line with* **average** 2-**distortion** $\mathcal{O}(\alpha(G)^2)$.

Proof Step 1: Non-expansive for shortest-path metric $\implies$ partially non-expansive for original L_2 metric $\implies$ adjacent balls still intersect!

Refining the Cheeger-Style Inequality

Lemma (New)

For a graph G with maximum degree Δ,

$$\gamma^{(n)}(G) \lesssim \gamma^{(1)}(G) \lesssim \gamma^{(n)}(G) \cdot \alpha(G)^2.$$

New method: Embeddings of shortest-path metrics!

Let $\omega(v) :=$ radius of v. Use ω-weighted SPs on G.

Theorem (Rab08, BLR10, KR10, unpublished follow-up work)

For a graph G with vertex-weights $\omega : V \to \mathbb{R}_{\geq 0}$, a deterministic algorithm can compute a **non-expansive** *embedding of d_ω into the line with* **average 2-distortion** $\mathcal{O}(\alpha(G)^2)$.

Proof Step 1: Non-expansive for shortest-path metric $\implies$ partially non-expansive for original L_2 metric $\implies$ adjacent balls still intersect!

Proof Step 2: Average 2-distortion bound $\implies$ normalizing denominator in objective only goes up by $\mathcal{O}(\alpha(G)^2)$.

Two kinds of upper bounds on $\gamma^{(n)}(G)$: Geometric and Combinatorial

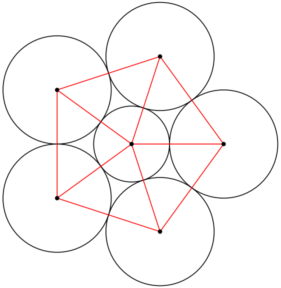
Reminder: $\gamma^{(n)}(G)$ small for a class of graphs $\implies$ small sparse cuts $\psi(S)$.

Two kinds of upper bounds on $\gamma^{(n)}(G)$: Geometric and Combinatorial

Reminder: $\gamma^{(n)}(G)$ small for a class of graphs $\implies$ small sparse cuts $\psi(S)$.

Geometric Bounds on $\gamma^{(n)}(G)$

Rich theory of **circle packings**!

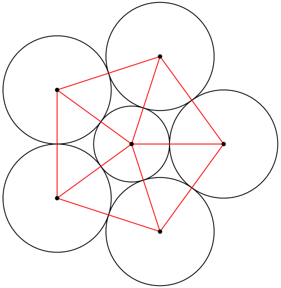


Two kinds of upper bounds on $\gamma^{(n)}(G)$: Geometric and Combinatorial

Reminder: $\gamma^{(n)}(G)$ small for a class of graphs $\implies$ small sparse cuts $\psi(S)$.

Geometric Bounds on $\gamma^{(n)}(G)$

Rich theory of **circle packings**!



Combinatorial Bounds on $\gamma^{(n)}(G)$

Rich theory of **congestion bounds** via crossing numbers!

E.g., crossing number lemma:

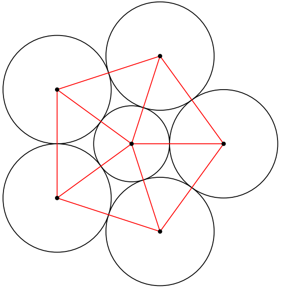
$$\mathrm{cr}(G) \gtrsim \frac{m^3}{n^2}.$$

Two kinds of upper bounds on $\gamma^{(n)}(G)$: Geometric and Combinatorial

Reminder: $\gamma^{(n)}(G)$ small for a class of graphs $\implies$ small sparse cuts $\psi(S)$.

Geometric Bounds on $\gamma^{(n)}(G)$

Rich theory of **circle packings**!



Combinatorial Bounds on $\gamma^{(n)}(G)$

Rich theory of **congestion bounds** via crossing numbers!

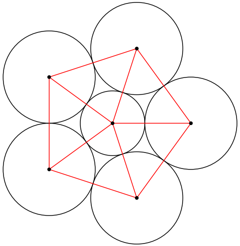
E.g., crossing number lemma:

$$\mathrm{cr}(G) \gtrsim \frac{m^3}{n^2}.$$

Either kind $\implies$ $\gamma^{(1)}(G) \lesssim \frac{1}{n}$ for G planar.

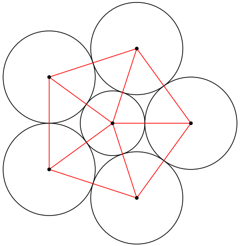
Geometric Bounds: Planar Case

Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.



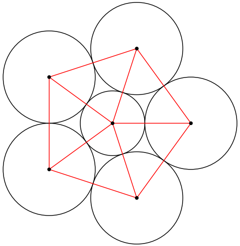
Geometric Bounds: Planar Case

Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.



Geometric Bounds: Planar Case

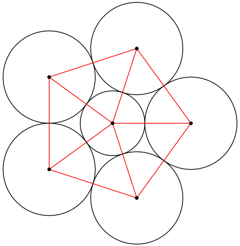
Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.



Step 2: Stereographic projection: Put circle packing onto unit sphere (centroid at origin).

Geometric Bounds: Planar Case

Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.

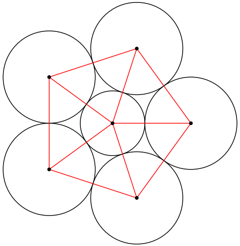


Step 2: Stereographic projection: Put circle packing onto unit sphere (centroid at origin).

Step 3: Total area of representation $\leq$ area of unit sphere (4π)

Geometric Bounds: Planar Case

Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.



Step 2: Stereographic projection: Put circle packing onto unit sphere (centroid at origin).

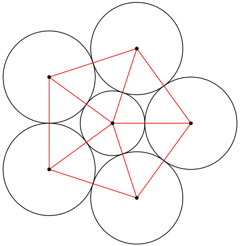
Step 3: Total area of representation $\leq$ area of unit sphere (4π)

Done!

Use $y(v) := 2 \cdot \mathrm{rad}(v)^2$,
$f(v) :=$ centre of disk on sphere.

Geometric Bounds: Planar Case

Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.



Step 2: Stereographic projection: Put circle packing onto unit sphere (centroid at origin).

Step 3: Total area of representation $\leq$ area of unit sphere (4π)

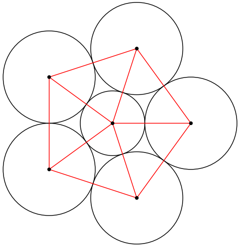
Done!

Use $y(v) := 2 \cdot \mathrm{rad}(v)^2$,
$f(v) :=$ centre of disk on sphere.

Centroid at origin:
$\sum_{v \in V} f(v) = \overline{0}$

Geometric Bounds: Planar Case

Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.



Step 2: Stereographic projection: Put circle packing onto unit sphere (centroid at origin).

Step 3: Total area of representation $\leq$ area of unit sphere (4π)

Done!

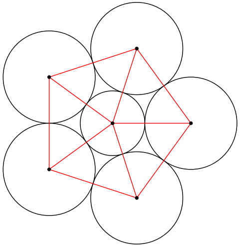
Use $y(v) := 2 \cdot \mathrm{rad}(v)^2$,
$f(v) :=$ centre of disk on sphere.

Centroid at origin:
$\sum_{v \in V} f(v) = \overline{0}$

Touching circles ($uv \in E$):
$y(u) + y(v) \geq (\mathrm{rad}(u) + \mathrm{rad}(v))^2$
$\geq \|f(u) - f(v)\|_2^2$

Geometric Bounds: Planar Case

Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.



Step 2: Stereographic projection: Put circle packing onto unit sphere (centroid at origin).

Step 3: Total area of representation $\leq$ area of unit sphere (4π)

Done!

Use $y(v) := 2 \cdot \mathrm{rad}(v)^2$,
$f(v) :=$ centre of disk on sphere.

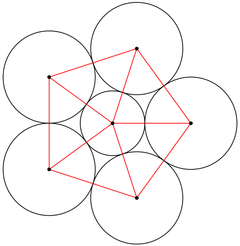
Centroid at origin:
$\sum_{v \in V} f(v) = \overline{0}$

Touching circles ($uv \in E$):
$y(u) + y(v) \geq (\mathrm{rad}(u) + \mathrm{rad}(v))^2$
$\geq \|f(u) - f(v)\|_2^2$

Unit sphere: $\sum_{x \in V} \|f(x)\|_2^2 = n$

Geometric Bounds: Planar Case

Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.



Step 2: Stereographic projection: Put circle packing onto unit sphere (centroid at origin).

Step 3: Total area of representation $\leq$ area of unit sphere (4π)

Done!

Use $y(v) := 2 \cdot \mathrm{rad}(v)^2$,
$f(v) :=$ centre of disk on sphere.

Centroid at origin:
$\sum_{v \in V} f(v) = \overline{0}$

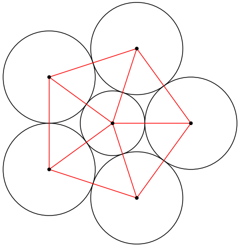
Touching circles ($uv \in E$):
$y(u) + y(v) \geq (\mathrm{rad}(u) + \mathrm{rad}(v))^2$
$\geq \|f(u) - f(v)\|_2^2$

Unit sphere: $\sum_{x \in V} \|f(x)\|_2^2 = n$

Area bound: $\sum_{v \in V} y(v) \lesssim 1$.

Geometric Bounds: Planar Case

Step 1: Planar circle packing theorem. Every planar graph admits touching circles representation.



Step 2: Stereographic projection: Put circle packing onto unit sphere (centroid at origin).

Step 3: Total area of representation $\leq$ area of unit sphere (4π)

Done!

Use $y(v) := 2 \cdot \mathrm{rad}(v)^2$,
$f(v) :=$ centre of disk on sphere.

Centroid at origin:
$\sum_{v \in V} f(v) = \overline{0}$

Touching circles $(uv \in E)$:
$y(u) + y(v) \geq (\mathrm{rad}(u) + \mathrm{rad}(v))^2 \geq \|f(u) - f(v)\|_2^2$

Unit sphere: $\sum_{x \in V} \|f(x)\|_2^2 = n$

Area bound: $\sum_{v \in V} y(v) \lesssim 1$.

Result: $$\frac{\sum_{v \in V} y(v)}{\sum_{x \in V} \|f(x)\|_2^2} \lesssim \frac{1}{n}$$

Combinatorial Bounds: Overview for Genus-g Graphs

$$\mathrm{cr}_g(K_n) \gtrsim \frac{n^4}{g}$$

$\downarrow$

$$\mathrm{con}_2^*(G) \gtrsim \frac{n^2}{\sqrt{g}}$$

$\downarrow$ Integrality gap

$$\mathrm{con}_2(G) \gtrsim \frac{n^2}{\sqrt{g}}$$

$\updownarrow$ Strong duality

$$\overline{s}_2(G) \gtrsim \frac{n^2}{\sqrt{g}}$$

Cauchy-Schwarz $\longrightarrow$

$$\sup_{\substack{\omega: V \to \mathbb{R}_{\geq 0} \\ ||\omega||_2 \leq 1}} \sum_{u,v \in V} [d_G^{\omega}(u,v)]^2 \gtrsim \frac{n^2}{g}$$

$\overline{s}_2(G)$ $\downarrow$ Metric embedding

$$\overline{Q}_1^{(1)}(G) \gtrsim \frac{1}{\sqrt{g}\log g}$$

Cauchy-Schwarz $\longrightarrow$

$\sup \ldots$ $\downarrow$ Metric embedding

$$\overline{Q}_2^{(1)}(G) \gtrsim \frac{n^2}{g(\log g)^2}$$

$\longleftrightarrow$

$$\gamma^{(1)}(G) \lesssim (\log g)^2 \frac{g}{n}$$

$\uparrow$ Relaxation

$$\gamma^{(n)}(G) \lesssim (\log g)^2 \frac{g}{n}$$

$\updownarrow$ Strong duality

$$\lambda_2^*(G) \lesssim (\log g)^2 \frac{g}{n}$$

Fin

Reweighted Spectral Partitioning Works:
A Simple Algorithm for Vertex Separators in Special Graph Classes

Jack Spalding-Jamieson

`https://arxiv.org/pdf/2506.01228`

Lots more results in the paper!

Fin

Reweighted Spectral Partitioning Works:
A Simple Algorithm for Vertex Separators in Special Graph Classes

Jack Spalding-Jamieson

https://arxiv.org/pdf/2506.01228

Lots more results in the paper!

- New separator theorems for some geometric graph classes.
- Other bounds on $\gamma^{(n)}$.
- A new bound on λ_2 for genus-g graphs.
- Fixes for a couple proofs from previous papers.

Fin

Reweighted Spectral Partitioning Works:
A Simple Algorithm for Vertex Separators in Special Graph Classes

Jack Spalding-Jamieson

https://arxiv.org/pdf/2506.01228

Lots more results in the paper!

- New separator theorems for some geometric graph classes.
- Other bounds on $\gamma^{(n)}$.
- A new bound on λ_2 for genus-g graphs.
- Fixes for a couple proofs from previous papers.

Questions?