Conflict-Based Local Search for Minimum Partition into Plane Subgraphs

CG Challenge 2022, Team `gitastrophe`

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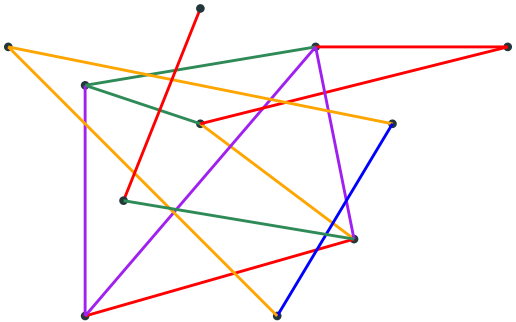
2022/06/09

Standings

Rank	Team	Junior team	Score
1	Shadoks		225.0
2	gitastrophe	✔	217.48574745772237
3	LASAOFOOFUBESTINNRRALLDECA		211.80303248033107
4	TU Wien	✔	195.9666148217582

Problem Statement

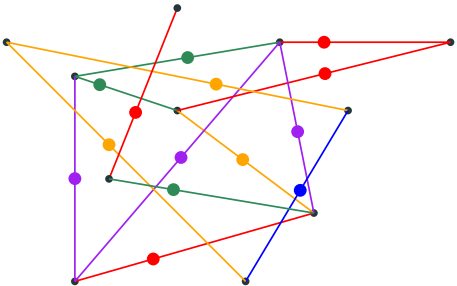
Input: A straight-line drawing of a graph $G = (V, E)$.
Output: A partition of G into plane subgraphs (COLOURS).
Goal: Minimize the number of subgraphs.

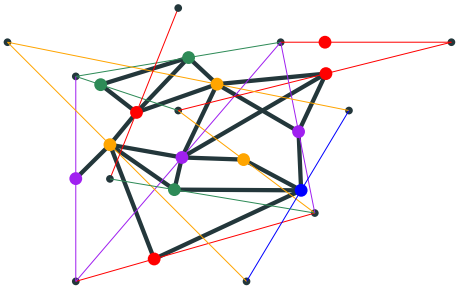


Reduction to Vertex Colouring

Construct a conflict graph G':

- $V(G') := E(G)$
- $E(G') :=$ the pairwise intersections of the straight-line edges.





Approach - Overview

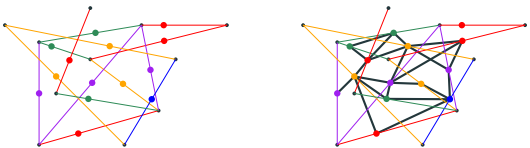
Two main components:

1. (Very basic) Initialization
2. Optimization
    - Conflict-Based Local Search
    - Alternative heuristics

Initialization

The simplest initialization strategy is:

- Start with all the edges uncoloured.
- Loop through the straight-line edges.
- For each one, colour it the minimum colour that doesn't conflict with any of its neighbours in the conflict graph.



Good orderings of the edges:

- Sorted by slope.
- Sorted by decreasing order of degree in the conflict graph (Welsh and Powell. 1967).

Optimization - Conflict-Based Local Search

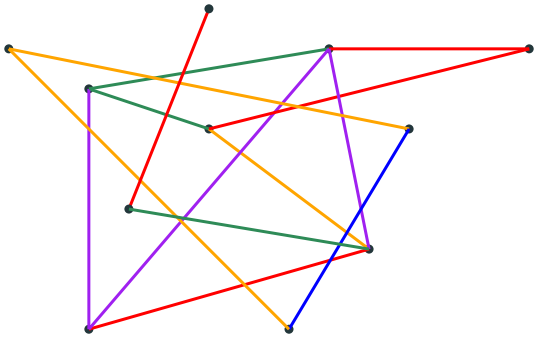
Conflict-Based Local Search/Conflict Optimization

- Initially used by was used by team Shadoks in CG:SHOP 2021 (Crombez et al. 2021).
- Very broad idea, can be applied this year as well.

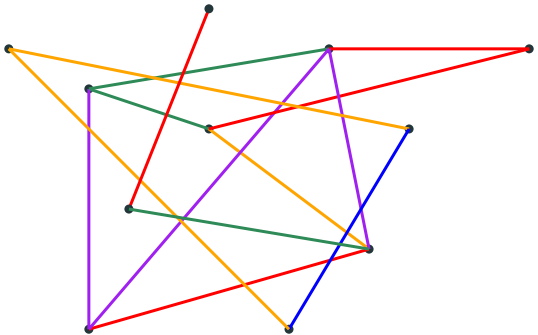
Main idea:

- Eliminate an entire colour class **without** giving the edges a new colour.
- Try to colour each uncoloured edge while minimizing a **conflict score**, i.e. a heuristic.
- Uncolour the conflicting edges when colouring the edge.

Optimization Example (1)

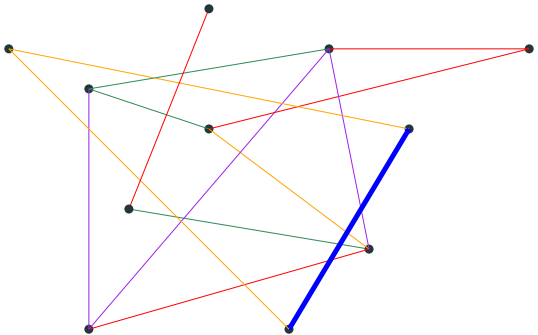


Optimization Example (4)



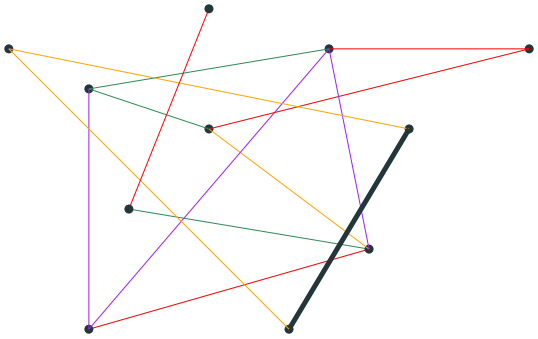
Step 1: Eliminate a Colour

Optimization Example (5)



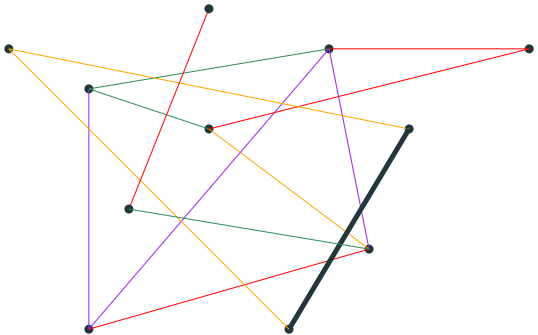
Choose blue to eliminate

Optimization Example (6)



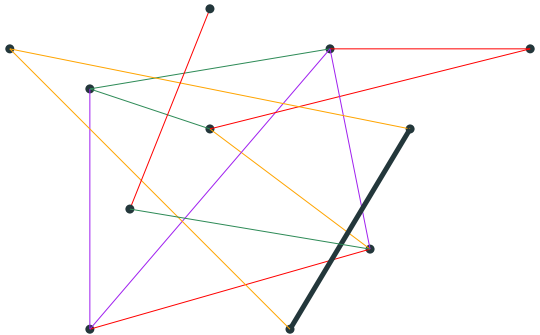
Uncolour all blue edges

Optimization Example (7)



Look at an uncoloured edge

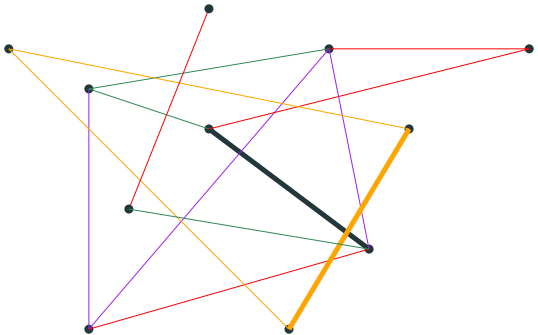
Optimization Example (8)



Pick a new colour according to a “conflict score” heuristic

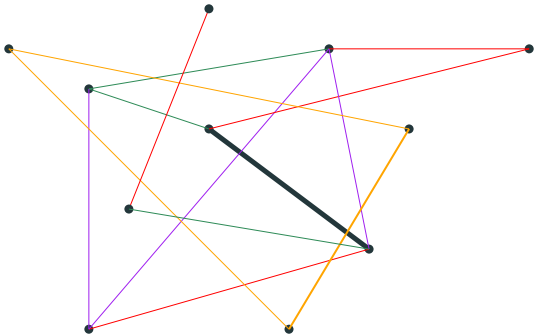
Choose orange

Optimization Example (9)



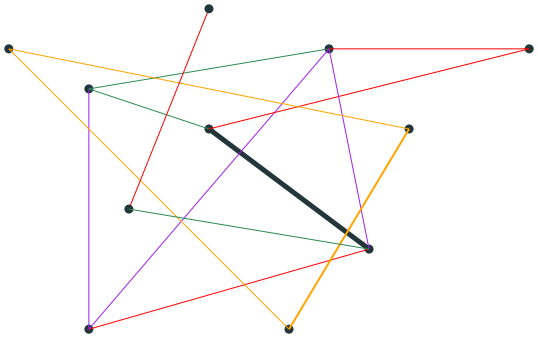
Colour the edge and uncolour all conflicting edges

Optimization Example (10)



If there is one: Look at an uncoloured edge

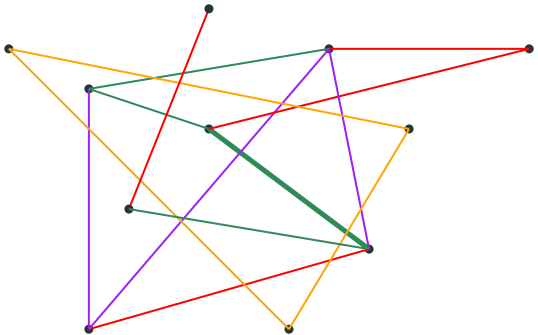
Optimization Example (11)



Pick a new colour according to a “conflict score” heuristic

Choose green

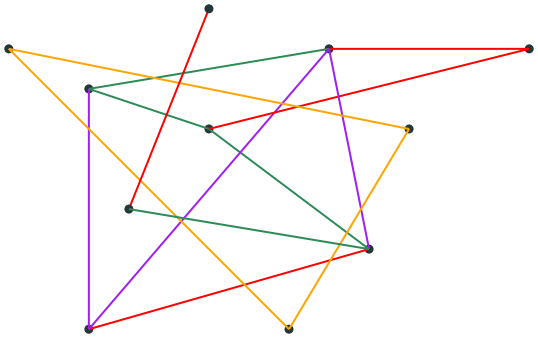
Optimization Example (12)



Pick a new colour according to a “conflict score” heuristic

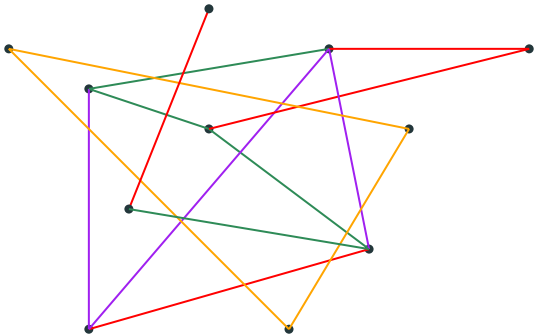
Choose green

Optimization Example (13)



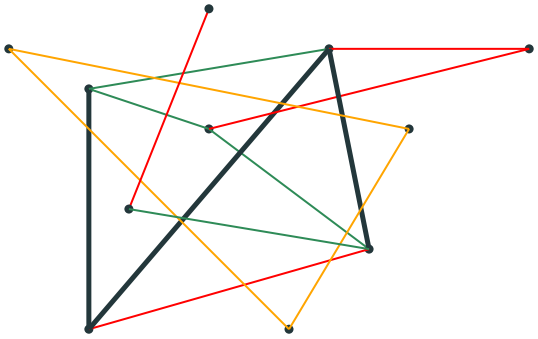
One colour down!

Optimization Example (14)



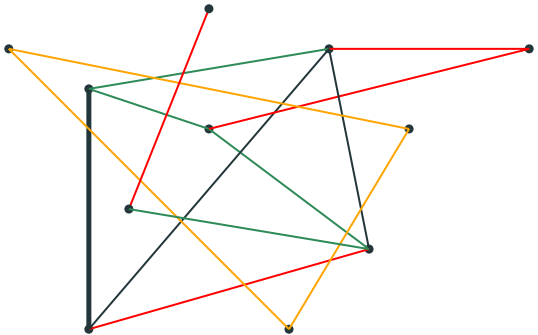
Let's try to eliminate another one: Purple

Optimization Example (15)



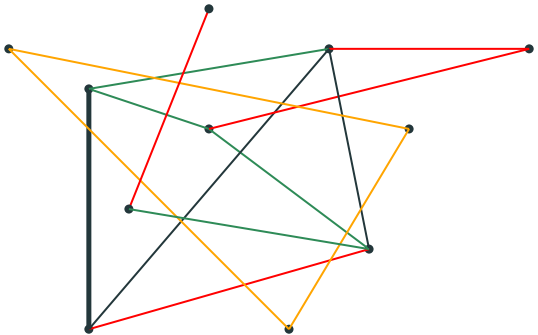
Uncolour all the purple edges

Optimization Example (16)



Look at an uncoloured edge

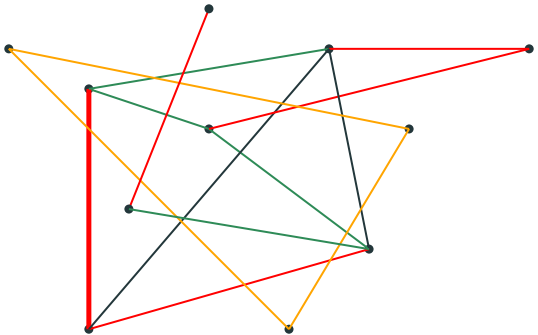
Optimization Example (17)



Choose a colour based on a “conflict score”

Choose red

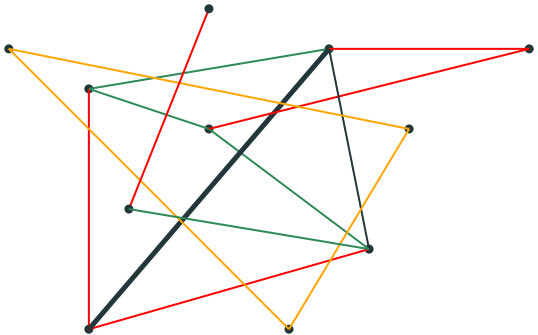
Optimization Example (18)



Colour the edge red and uncolour any conflicting edges

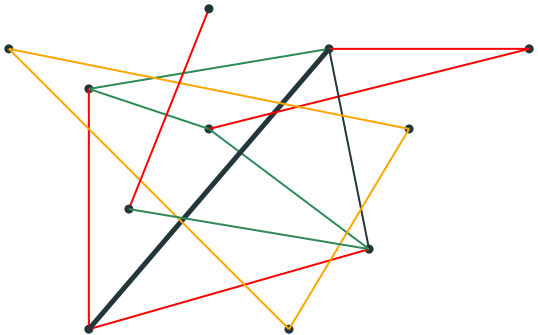
(none in this case)

Optimization Example (19)



Look at another uncoloured edge

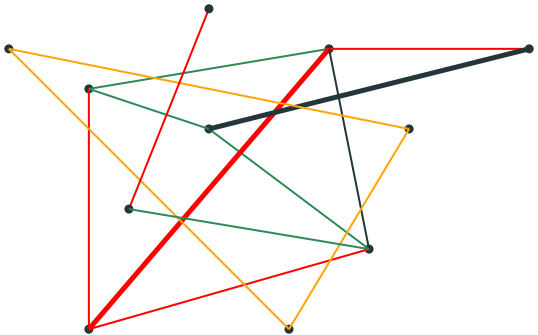
Optimization Example (20)



Choose a colour for it based on conflict score

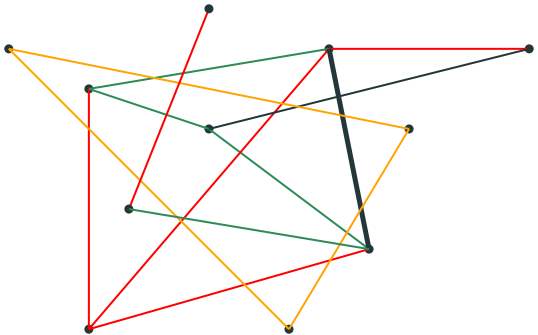
Choose red

Optimization Example (21)



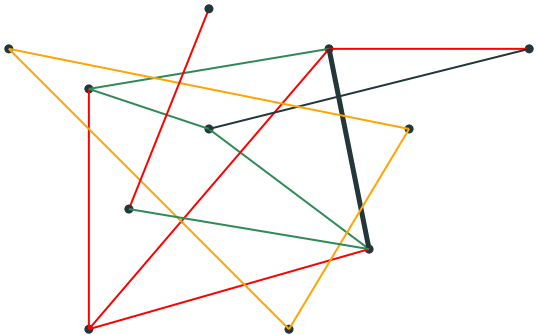
Colour the edge red and uncolour any conflicting edges

Optimization Example (22)



Look at an uncoloured edge

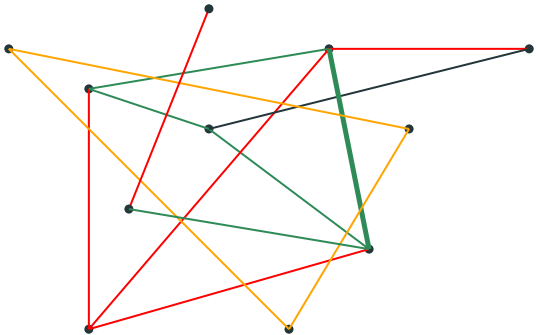
Optimization Example (23)



Choose a colour based on a “conflict score”

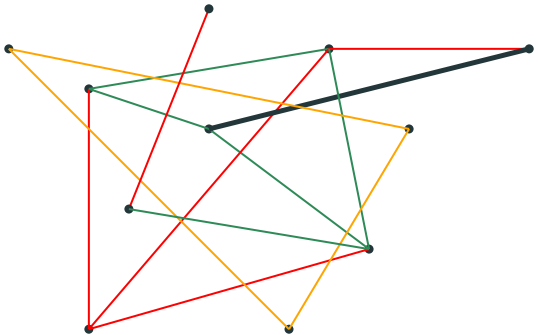
Choose green

Optimization Example (24)



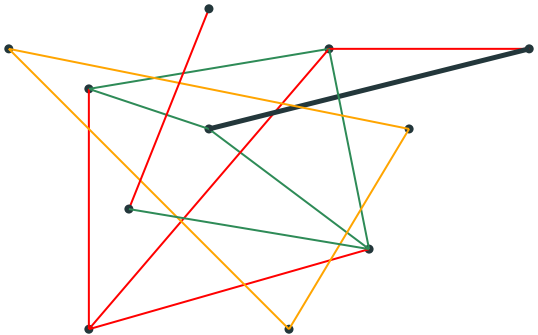
Colour the edge green and uncolour any conflicting edges

Optimization Example (25)



Look at an uncoloured edge

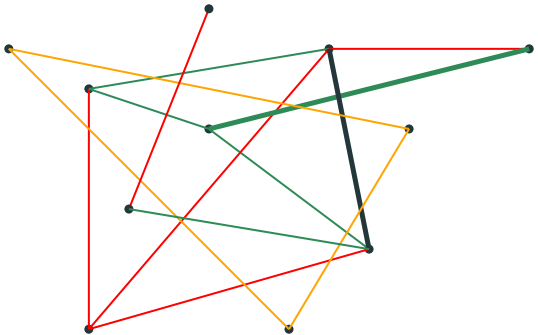
Optimization Example (26)



Choose a colour based on a “conflict score”

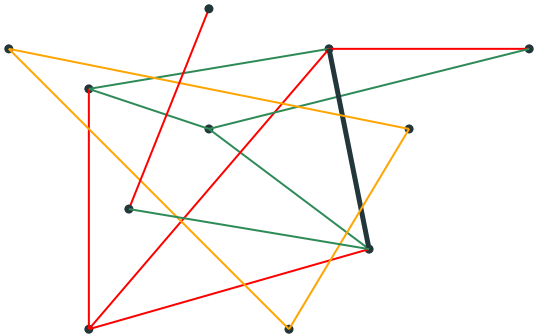
Choose green

Optimization Example (27)



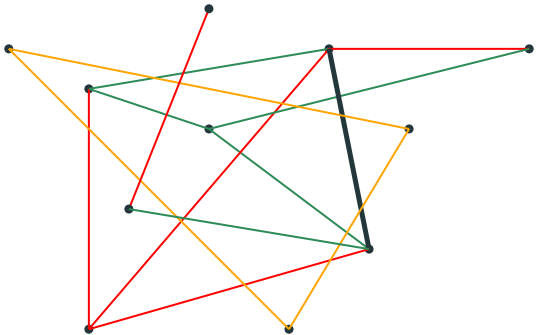
Colour the edge green and uncolour any conflicting edges

Optimization Example (28)



Look at an uncoloured edge

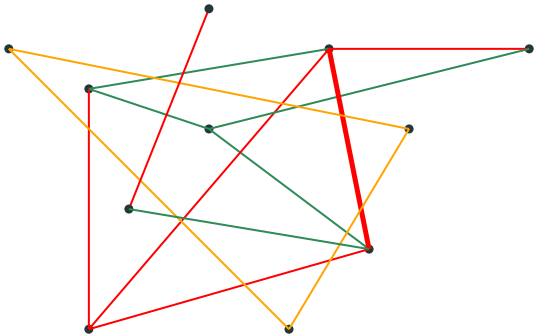
Optimization Example (29)



Choose a colour based on a “conflict score”

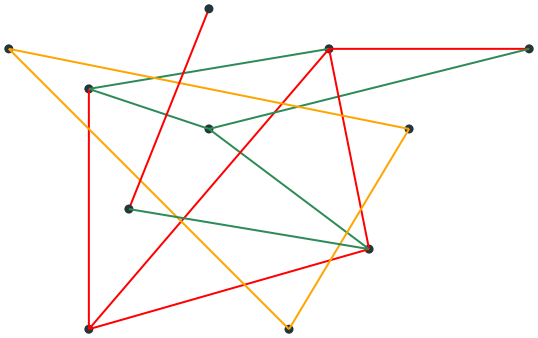
Choose red

Optimization Example (30)



Colour the edge red and uncolour any conflicting edges

Optimization Example (31)



Done!

Optimization - Heuristics

Conflict score:

$$\sum_{\substack{e' \in C_i \\ (e',e) \in E(G')}} 1 + q(e')^2$$

$q(e')$ is the number of times e' was uncoloured.

Optimization - Heuristics

Conflict score:

$$\sum_{\substack{e' \in C_i \\ (e',e) \in E(G')}} 1 + q(e')^2$$

$q(e')$ is the number of times e' was uncoloured.

Alternative:

$$\sum_{\substack{e' \in C_i \\ (e',e) \in E(G')}} 1$$

Comparison to Vertex Colouring Approaches

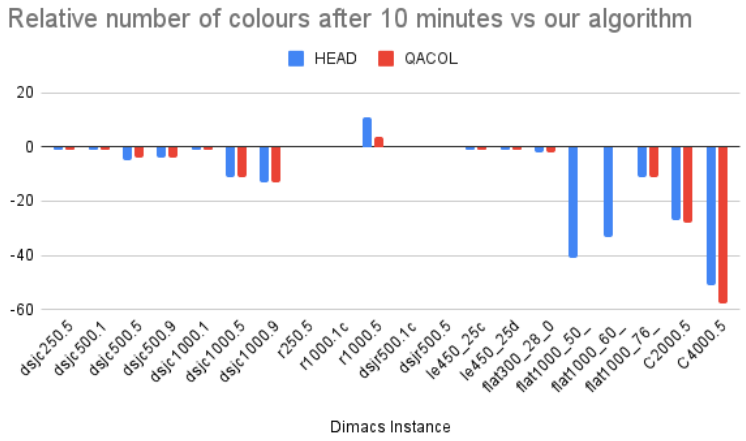


Figure 1: 10 minutes of our algorithm versus standard approaches on dimacs graph colouring instances.

Thank you for listening

Thank you organizers for hosting the challenge!

Our code is available at: `https://github.com/jacketsj/cgshop2022-gitastrophe`